

# Test of the Einstein equivalence principle with spectral distortions in the cosmic microwave background

Shun Arai<sup>1,\*</sup>, Daisuke Nitta<sup>1</sup>, and Hiroyuki Tashiro<sup>1</sup>

<sup>1</sup>*Department of Physics and Astrophysics, Nagoya University, Nagoya 464-8602, Japan*

(Dated: December 6, 2016)

The Einstein equivalence principle (EEP) can be verified by the measurement of the spectral distortions of the cosmic microwave background (CMB). One of the consequences of the EEP on cosmological scales is the energy independency of the cosmological redshift effect. We propose a new test of the energy independency of the redshift effect by the measurement of the spectral distortion of CMB. In general relativity, the energy independency of the redshift effect is ensured by the Friedmann-Robertson-Walker (FRW) metric which does not depend on energy. We show that the CMB spectral distortions arise when the FRW metric has the energy dependence. Assuming the simple energy-dependent form of the FRW metric, we evaluate the CMB distortions. From the COBE/FIRAS bound, we find that the deviation degree from the EEP is, at least, less than  $10^{-5}$  at the CMB energy scales.

PACS numbers: 04.80.Cc, 95.30.Sf, 98.70.Vc, 98.80.Es

Keywords: General Relativity, Einstein Equivalence Principle, Cosmic Microwave Background (CMB)

## I. INTRODUCTION

Observations of the cosmic microwave background (CMB) have become essential tools in modern cosmology. Precise measurements of the CMB temperature and polarization anisotropies provide valuable information about the Universe [1]. Recently, the measurement of CMB spectral distortions, that is, the deviation of the CMB frequency spectrum from a blackbody spectrum, has been expected as a new cosmological probe.

COBE/FIRAS has obtained the almost perfect blackbody spectrum of the CMB with the temperature  $T_0 = 2.726$  K [2]. Although they have not been detected yet, CMB distortions can be generated within the standard cosmological model as well as with new physics (for reviews, see [3–5]). Currently, the observational bound on the spectral distortions is given in terms of two types of distortions,  $\mu$  and  $y$ -type distortions [6, 7]. The  $\mu$ -type distortion is described with a nonvanishing chemical potential  $\mu$  and created at  $10^6 \gtrsim z \gtrsim 5 \times 10^4$  where, even if the CMB spectral distortions arise, Compton scattering is efficient enough to maintain the kinetic equilibrium of CMB photons. The  $y$ -type distortion is parametrized by the Compton  $y$  parameter and generated in lower redshifts  $z < 5 \times 10^4$  where, once the CMB spectrum is distorted, the kinetic equilibrium of CMB photons is no longer maintained. The current constraints on the distortion parameters are provided by COBE/FIRAS as  $|y| < 1.5 \times 10^{-5}$  and  $|\mu| < 9 \times 10^{-5}$  [8]. To improve these bounds, next-generation CMB spectrometers are being discussed [9, 10]. The future measurements or constraints on the CMB distortions allow us to access the properties of primordial fluctuations [11], the nature of dark matter [12], the abundance of primordial black

holes [13], the existence of primordial magnetic fields [14] and other high-energy physics [15]. In this paper, we discuss that the measurement of CMB distortions can also test general relativity (GR), in particular, the Einstein equivalence principle (EEP).

Since GR was proposed as the theory of gravity by Einstein, the theory has passed almost all tests such as ground-based and Solar System experiments [16]. And furthermore, the gravitational wave detection by LIGO proves the accuracy of the theory even in a strong gravitational field [17, 18]. However, it still leaves room to verify GR at the cosmological scales. Since the first evidence was presented by the type-Ia supernova observations [19, 20], independent cosmological observations strongly support the accelerating expansion of the Universe. As an origin of this acceleration, GR requires the existence of unknown dark energy. Alternatively, the modification of GR on cosmological scales is suggested to explain the acceleration as an effect of gravity [21–23]. Therefore, it is still quite important to verify GR in the cosmological context, and we pay attention to the validity of the EEP which is one of the fundamental principles in GR.

The EEP is tested from laboratory to Solar System scales by many authors (for reference, see Ref. [24]). In these studies, the validity of the EEP has been obtained from the travels of a test particle through the gravitational potential. Therefore, as the constraint on the EEP, these studies have provided the constraint on the energy dependency of parametrized-post-Newtonian (PPN) parameter  $\gamma$ . Recently, this energy dependency has been also tested by using high-energy photons emitted by gamma ray bursts, fast radio bursts, and TeV blazars with the gravitational potential of the Milky Way [25–32].

In this paper, we focus on the independency of the cosmological redshift effect on the energy of a test particle. Because this energy independency is one of the conse-

---

\* arai.shun@a.mbox.nagoya-u.ac.jp

quences of the EEP on cosmological scales, it is important to test the independency of the redshift effect by cosmological observations. We show that the independency of the redshift effect can be verified by measurement of the CMB distortion. After submitting our paper, Ref. [33] appeared. They have investigated the energy dependence of the cosmological redshift effect using the emission lines over the 3700–6800 Å range in SDSS spectroscopic data at  $0.1 < z < 0.25$ . Their conclusion is that they cannot find any energy-dependence of the redshift with a precision of  $10^{-6}$  at  $z < 0.1$  and  $10^{-5}$  at  $0.1 < z < 0.25$ . Our method is complementary with theirs because probing energy is different. Besides, CMB distortion can verify the EEP up to redshifts larger than  $z \sim 1000$ .

To demonstrate the test of the EEP through the CMB distortion, we introduce a simple energy dependence of the Friedmann-Robertson-Walker (FRW) metric. Generally, when a metric depends on energy, the EEP is violated in this metric theory of gravity. In other words, the existence of the energy dependence of the metric means that the structure of spacetime felt by a test particle depends on its own energy.

In GR, although CMB photons are redshifted due to the cosmic expansion, the blackbody spectrum of the CMB is hold during their free-streaming because the EEP ensures that redshift effect is independent of the photon energy. However, when the redshift effect depends on the photon energy, the deviation from the blackbody spectrum arises even in the free streaming regime. We evaluate the CMB distortion and obtain the constraints on the accuracy of the EEP on cosmological time and length scales through a comparison with the COBE/FIRAS data.

## II. ENERGY-DEPENDENT FRW METRIC

Since the energy dependency of the metric violates the EEP, we first consider the energy-dependent FRW metric. Taking into account the cosmological principle, we can be allowed to introduce two energy dependent functions,  $f(E)$  and  $g(E)$ , in the FRW metric as

$$ds^2 = -\frac{dt^2}{f^2(E)} + \frac{a^2(t)}{g^2(E)} \delta_{ij} dx^i dx^j, \quad (1)$$

where  $E$  denotes the energy of a photon observed by a free-falling observer in this metric, and  $f(E)$  and  $g(E)$  are arbitrary functions of  $E$ . Although, without assuming any certain gravity theory, we determine the form of Eq. (1) based on the cosmological principle, the same energy dependence is often discussed in rainbow gravity, which is one of the gravity theories without the EEP [34–36].

Since the FRW metric depends on the energy, the redshift effect due to the cosmological expansion also has energy dependence. To derive the redshift effect, we consider the geodesic equation for a photon with energy  $E$ .

In the metric given by Eq. (1), nonvanishing Christoffel symbols are

$$\begin{aligned} \Gamma_{00}^0 &= -\frac{\dot{f}}{f}, & \Gamma_{ij}^0 &= \left(\frac{f}{g}\right)^2 \left(a\dot{a} - a^2 \frac{\dot{g}}{g}\right) \delta_{ij}, \\ \Gamma_{0j}^i &= \left(\frac{\dot{a}}{a} - \frac{\dot{g}}{g}\right) \delta_j^i, \end{aligned} \quad (2)$$

where the dot denotes the derivative with respect to time.

Therefore, the geodesic equation provides the modified redshift effect,

$$\dot{E} = -\frac{\dot{a}}{a} \left(1 - \frac{d \log g}{d \log E}\right)^{-1} E. \quad (3)$$

When  $f$  and  $g$  are constant, the redshift effect is the same as in GR.

## III. CMB DISTORTIONS DUE TO THE ENERGY-DEPENDENT REDSHIFT EFFECT

After the epoch of recombination, the universe becomes transparent for photons and they are free to stream out. During such a free-streaming regime, the evolution of the CMB photon energy distribution is given by the collisionless Boltzmann equation. Assuming the homogeneity and isotropy of the Universe, the collisionless Boltzmann equation in the metric by Eq. (1) can be described as

$$\frac{\partial n_E}{\partial t} - \frac{\dot{a}}{a} E \left(1 - \frac{d \log g}{d \log E}\right)^{-1} \frac{\partial n_E}{\partial E} = 0. \quad (4)$$

Although the general solution of Eq. (4) is provided in a function of the combination value,  $aE/g$ , we need the initial condition of the energy distribution to solve Eq. (4).

Well before the epoch of recombination, the time scale of thermal equilibrium for CMB photons is much shorter than the cosmological time scale. In this regime, when the deviation from a blackbody spectrum arises, the deviation is quickly erased and the blackbody spectrum is maintained. Therefore, for simplicity, we assume that the energy distribution of the CMB is a blackbody spectrum,  $[\exp(E/T_{\text{re}}) - 1]^{-1}$ , at the epoch of recombination, where  $T_{\text{re}}$  is the temperature at that epoch. However, as mentioned above, CMB distortions can be generated below  $z \sim 10^6$ , which is well before the epoch of the recombination. During this regime, the evolution of the CMB distortions is provided by the collisional Boltzmann equation. We will discuss this issue later.

With this assumption, the solution of Eq. (4) is given by

$$n_E = \frac{1}{\exp[\eta(E, z)E/T_z] - 1}, \quad (5)$$

where  $T_z = T_{\text{re}}(1+z)/(1+z_{\text{re}})$  with the redshift for the epoch of recombination  $z_{\text{re}}$ , and  $\eta(E, z)$  is provided by

$$\eta(E, z) = \frac{g(E_{\text{re}}(E, z))}{g(E)}, \quad (6)$$

where the function  $E_{\text{re}}(E, z)$  represents the energy at  $z_{\text{re}}$  for a photon whose energy is redshifted to  $E$  at the redshift  $z$ . We can obtain  $E_{\text{re}}(E, z)$  from Eq. (3). Since various tests support the validity of GR, we assume that  $g^{-1}$  can be approximated in  $g^{-1} \approx 1 + h(E)$  with  $h(E) \ll 1$ . In the leading order of  $h(E)$ , the function  $\eta(E, z)$  can be expanded in

$$\eta(E, z) \approx 1 + h(E) - h\left(\frac{1 + z_{\text{re}}}{1 + z}E\right). \quad (7)$$

The aim of this paper is to obtain the constraint on  $h(E)$  from the measurement of the CMB distortions. Here we demonstrate two simple cases of the function  $h(E)$ . In the first case,  $h(E)$  is a linear function of  $E$ . In the second case,  $h(E)$  is proportional to  $E^{-1}$ .

#### A. The case with $h(E) \propto E$

We assume that the form of  $h(E)$  is given by

$$h(E) = \delta_{T_0} E/T_0, \quad (8)$$

with  $\delta_{T_0} \ll 1$ . Here the parameter  $\delta_{T_0}$  represents the deviation degree from the EEP at the energy scale  $T_0$ . The CMB photon energy distribution at the present epoch is given from Eqs. (5) and (7). Expanding the photon energy distribution up to the linear order of  $\delta_{T_0}$ , we obtain

$$n_E \approx \frac{1}{\exp(E/T_0) - 1} + \frac{\exp(E/T_0)}{[\exp(E/T_0) - 1]^2} \left(\frac{E}{T_0}\right)^2 z_{\text{re}} \delta_{T_0}. \quad (9)$$

The first term represents the blackbody spectrum with  $T_0$  and the second term provides the deviation from the blackbody spectrum.

We define the relative deviation from the blackbody spectrum as  $\Delta_E = (n_E - n_{\text{BB},E})/n_{\text{BB},E}$  where  $n_{\text{BB},E}$  is the blackbody spectrum with  $T_0$ . According to Eq. (9)  $\Delta_E$  is given by

$$\Delta_E = \frac{\exp(E/T_0)}{\exp(E/T_0) - 1} \left(\frac{E}{T_0}\right)^2 z_{\text{re}} \delta_{T_0}. \quad (10)$$

We show  $\Delta_E$  as a function of  $E$  in Fig. 1. Here we set  $\delta_{T_0} = 10^{-9}$  and  $z_{\text{re}} = 1100$ . COBE/FIRAS has provided the possible residual from the blackbody spectrum [8]. We plot the residual as blue points in Fig. 1. From the figure, we conclude that COBE/FIRAS gives the upper bound,

$$|\delta_{T_0}| \lesssim 10^{-9}. \quad (11)$$

#### B. The case with $h(E) \propto E^{-1}$

Next we consider the case where  $h(E)$  is represented as

$$h(E) = \delta_{T_0} T_0/E. \quad (12)$$

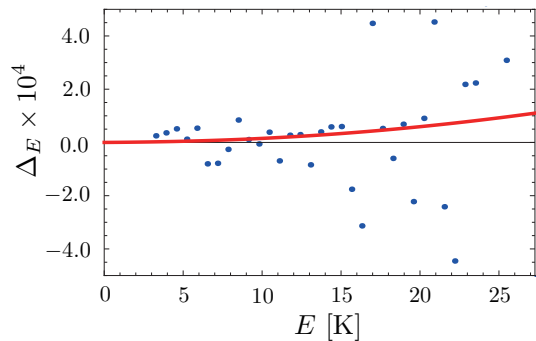


FIG. 1. The relative deviation from the blackbody spectrum,  $\Delta_E$ . Here we adopt  $h(E) = \delta_{T_0} E/T_0$  with  $\delta_{T_0} = 10^{-9}$ . The  $x$  axis is the energy of CMB photons in units of Kelvin. Blue points represent the residual measured by COBE/FIRAS [8].

Similarly to the previous case, we can obtain the CMB photon distribution from Eqs. (5) and (7). The CMB photon distribution can be approximated to

$$n_E \approx \left( \exp \left[ \frac{E}{T_0} \left( 1 + \frac{z_{\text{re}}}{1 + z_{\text{re}}} \frac{T_0}{E} \delta_{T_0} \right) \right] - 1 \right)^{-1}. \quad (13)$$

This corresponds to the Bose-Einstein distribution,  $n_E = (\exp(E/T_0 + \mu) - 1)^{-1}$ , with the dimensionless chemical potential  $\mu = z_{\text{re}} \delta_{T_0}/(1 + z_{\text{re}})$ .

COBE/FIRAS provides the constraint on  $\mu$  for CMB photons,  $|\mu| < 9 \times 10^{-5}$ . Therefore we obtain the limit,

$$|\delta_{T_0}| \lesssim 9 \times 10^{-5}. \quad (14)$$

Currently, PIXIE is designed to be 3 orders of magnitude better than COBE/FIRAS in the sensitivity [9]. The sensitivity of PIXIE is expected to be close to that required to measure the distortions arising from the dissipation of the scale-invariant primordial fluctuations,  $\mu \sim 10^{-8}$ , which is one of unavoidable cosmological sources for CMB distortions. When PIXIE provides the constraint  $\mu \lesssim 10^{-8}$ , the constraint on the EEP reaches  $|\delta_{T_0}| \lesssim 10^{-7}$  in the case of Eq. (12).

## IV. CONCLUSIONS

In this paper, we have proposed that the measurement of CMB spectral distortions can test the accuracy of the EEP on cosmological scales. The energy independence of the EEP is one of consequences of the EEP. When the FRW metric has energy dependence, the EEP is violated on cosmological scales. As a result, the geodesic equation of a photon is modified and the redshift effect depends on its energy. We have shown that, in the energy-dependent FRW metric, CMB distortions are generated even in the free-streaming regime through the energy-dependent redshift effect. The shape and amplitude of the distortion depends on the form of the energy dependency.

To parametrize the validity of the EEP in the FRW metric, we have introduced the deviation parameter  $\delta_{T_0}$  representing the deviation from the EEP on the CMB energy scale. We have analytically evaluated the CMB distortions in two simple power-law cases of the energy-dependent deviation in the FRW metric with the power law indices  $n = 1$  and  $n = -1$ . In the first case with  $n = 1$ , we have found that the COBE/FIRAS bound indicates that the EEP is valid within the degree of the deviation,  $|\delta_{T_0}| \lesssim 10^{-9}$ , on the CMB energy scale, 0.0001–1 eV. When  $n > 0$ , the deviations at higher energy scales are larger than at lower energy scales. This means that, as  $n$  becomes larger, the deviation increases at higher redshifts. Therefore, the constraint on  $\delta_{T_0}$  becomes tighter when  $n$  increases. In the second case with  $n = -1$ , the generated distortion is represented as the  $\mu$ -type distortion and the COBE/FIRAS bound provides the constraint  $|\delta_{T_0}| \lesssim 10^{-5}$ . When  $n < 0$ , the deviations at lower energy scales are larger than at higher energy scales. Therefore, we obtain  $|\delta_{T_0}| \lesssim 10^{-5}$  for  $n < 0$ . Depending on the energy dependence of the FRW metric, the spectral shape is different from the ordinary CMB distortions,  $\mu$ - and  $y$ -type distortions. Therefore, the precise measurement of the distortion shape can provide us with a strong constraint on the EEP violation.

It is worth summarizing previous works about the test of the EEP and providing comments on the relevance of our study. In previous studies, the accuracy of the EEP is investigated with the energy dependency of the PPN parameter  $\gamma$ . Using the gamma ray observations, the constraint is provided as  $\gamma_{\text{GeV}} - \gamma_{\text{MeV}} \lesssim 10^{-8}$  and  $\gamma_{\text{eV}} - \gamma_{\text{MeV}} \lesssim 10^{-7}$  [27]. In the radio frequency range, the energy difference of  $\gamma$  is less than  $10^{-8}$ , which is comparable to our results, from the observations of fast radio bursts [28, 32]. Since the constraints on the energy difference of  $\gamma$  is related to the gravitational potential, these constraints are valid for the Schwarzschild metric. Therefore, these constraints cannot be directly applicable to the FRW metric without taking a theory of gravity. In Ref. [37], the authors have discussed that the measurement of CMB distortions can provide the constraint on the time variation of the fine structure constant due to the EEP violation in the electromagnetic sector. Our constraint is completely independent of these limits. In more detail, we have provided a bound on the EEP in the FRW metric for the cosmological time scale from the epoch of recombination to the present time. Additionally, upcoming observations with PIXIE provide 3 orders of magnitude stronger constraints than that of COBE/FIRAS. Recently Ref. [33] has investigated the energy dependence of the cosmological redshift

with SDSS data. Their result is consistent with no energy dependence of the redshift effect with a precision of  $10^{-6}$  at  $z < 0.1$  and  $10^{-5}$  at  $0.1 < z < 0.25$ . In this work, they used the spectral lines over the 3700–6800 Å range whose energy range is higher than in the CMB observation frequencies. Although their investigated redshifts are not so high, their results are complementary with our results. According to both results, the violation of the EEP in the FRW metric is not found in the range from microwave to optical frequencies in the current observation precision.

In this paper, we have demonstrated that the measurement of the CMB distortions can test the EEP in the FRW metric by taking some assumptions. In particular, to evaluate the CMB distortions analytically, we neglected the evolution of the CMB distortions before the epoch of recombination. Although the distortions can be generated in the energy-dependent FRW metric before this epoch, it is necessary to solve the collisional Boltzmann equation numerically. Because the next-generation CMB spectrometers are being planned to measure CMB distortions precisely, further detailed calculation is required. We will address these issues for the EEP bound in our future works.

The spectral distortions of the CMB can be generated by other physical mechanisms, in particular, the processes related to the thermal history of the Universe. Therefore, it is difficult to solve these degeneracies to point out the effect of the EEP violation by only the CMB distortion measurement. However, although we have only studied the CMB distortions of the CMB, neutrinos and gravitons also suffer an energy-dependent redshift effect and their spectra are modified from ones in the standard cosmology when the EEP is violated in the FRW metric. Therefore, the frequency spectral measurement of not only CMB but also neutrinos and gravitational waves can allow us to obtain the observational suggestion to the EEP violation.

## ACKNOWLEDGMENTS

We thank Naoshi Sugiyama, Yuko Urakawa, and Jens Chluba for the useful discussions. S.A. and D.N. are in part supported by the Ministry of Education, Culture, Sports, Science and Technology, Japan (MEXT) Grant-in-Aid for Scientific Research on Innovative Areas, No. 15H05890, H.T. is supported by Japan Society for the Promotion of Science (JSPS) KAKENHI Grant No. 15K17646 and MEXT's Program for Leading Graduate Schools “Ph.D. professionals, Gateway to Success in Frontier Asia.”

---

[1] R. Adam *et al.* (Planck Collaboration), Planck 2015 results. I. Overview of products and scientific results, *As-*

*tron. Astrophys.* **594**, A1 (2016).

[2] D.J. Fixsen, The temperature of the cosmic microwave



- background, *Astrophys. J.* **707**, 916 (2009).
- [3] J. Chluba, and R.A. Sunyaev, The evolution of CMB spectral distortions in the early Universe, *Mon. Not. R. Astron. Soc.* **419**, 1294 (2012).
- [4] R. A. Sunyaev and R. Khatri, Unavoidable CMB spectral features and blackbody photosphere of our Universe, *Int. J. Mod. Phys. D* **22**, 1330014 (2013).
- [5] H. Tashiro, *Prog. Theor. Exp. Phys.* **2014**, 06B107 (2014).
- [6] Y. B. Zeldovich and R. A. Sunyaev, The interaction of matter and radiation in a hot-model universe, *Astrophys. Space. Sci.* **4**, 301 (1969).
- [7] R.A. Sunyaev, and Y.B. Zeldovich, *Astrophys. Space. Sci.* **7**, 20 (1970).
- [8] D.J. Fixsen, E.S. Cheng, J.M. Gales, J.C. Mather, R.A. Shafer, and E.L. Wright, *Astrophys. J.* **473**, 576 (1996).
- [9] A. Kogut *et al.*, The Primordial Inflation Explorer (PIXIE): A nulling polarimeter for cosmic microwave background observations, *J. Cosmol. Astropart. Phys.* 07 (2011) 025.
- [10] P. André *et al.*, PRISM (Polarized Radiation Imaging and Spectroscopy Mission): An extended white paper, *J. Cosmol. Astropart. Phys.* 02 (2014) 006.
- [11] R. A. Sunyaev and Y. B. Zeldovich, Small scale fluctuations of relic radiation, *Astrophys. Space Sci.* **7**, 3 (1970); W. Hu, D. Scott and J. Silk, Power spectrum constraints from spectral distortions in the cosmic microwave background, *Astrophys. J.* **430**, L5 (1994); W. Hu and N. Sugiyama, Thermal history constraints on the isocurvature baryon model, *Astrophys. Phys. J.* **436**, 456 (1994); J. Chluba, R. Khatri and R. A. Sunyaev, CMB at 2x2 order: The dissipation of primordial acoustic waves and the observable part of the associated energy release, *Mon. Not. R. Astron. Soc.* **425**, 1129 (2012); J.B. Dent, D.A. Easson and H. Tashiro, Cosmological constraints from CMB distortion, *Phys. Rev. D* **86**, 023514 (2012); E. Pajer and M. Zaldarriaga, A New Window on Primordial non-Gaussianity, *Phys. Rev. Lett.* **109**, 021302 (2012); J. Chluba, A.L. Erickcek and I. Ben-Dayan, Probing the inflaton: Small-scale power spectrum constraints from measurements of the CMB energy spectrum, *Astrophys. J.* **758**, 76 (2012); J. Chluba and D. Grin, CMB spectral distortions from small-scale isocurvature fluctuations, *Mon. Not. Roy. Astron. Soc.* **434**, 1619 (2013); M. Biagetti, H. Perrier, A. Riotto and V. Desjacques, Testing the running of non-Gaussianity through the CMB  $\mu$ -distortion and the halo bias, *Phys. Rev. D* **87**, 063521 (2013); A. Ota, T. Takahashi, H. Tashiro and M. Yamaguchi, CMB  $\mu$  distortion from primordial gravitational waves, *J. Cosmol. Astropart. Phys.* 10 (2014) 029 .
- [12] W. Hu and J. Silk, Thermalization constraints and spectral distortions for massive unstable relic particles, *Phys. Rev. Lett.* **70**, 2661 (1993); P. McDonald, R. J. Scherrer and T. P. Walker, Cosmic microwave background constraint on residual annihilations of relic particles, *Phys. Rev. D* **63**, 023001 (2001); J. Chluba, Could the Cosmological Recombination Spectrum Help Us Understand Annihilating Dark Matter?, *Mon. Not. Roy. Astron. Soc.* **402**, 1195 (2010); J. Chluba, Distinguishing different scenarios of early energy release with spectral distortions of the cosmic microwave background, *Mon. Not. Roy. Astron. Soc.* **436**, 2232 (2013); H. Tashiro, K. Kadota and J. Silk, Effects of dark matter-baryon scattering on redshifted 21 cm signals, *Phys. Rev. D* **90**, 083522 (2014); Y. Ali-Haïmoud, J. Chluba, and M. Kamionkowski, Constraints on Dark Matter Interactions with Standard Model Particles from Cosmic Microwave Background Spectral Distortions, *Phys. Rev. Lett.* **115**, 071304 (2015).
- [13] H. Tashiro and N. Sugiyama, Constraints on Primordial Black Holes by Distortions of Cosmic Microwave Background, *Phys. Rev. D* **78**, 023004 (2008); M. Ricotti, J. P. Ostriker and K. J. Mack, Effect of Primordial Black Holes on the Cosmic Microwave Background and Cosmological Parameter Estimates, *Astrophys. J.* **680**, 829 (2008);
- [14] K. Jedamzik, V. Katalinić, and A. V. Olinto, A Limit on primordial small scale magnetic fields from CMB distortions, *Phys. Rev. Lett.* **85**, 700 (2000); K. E. Kunze and E. Komatsu, Constraining primordial magnetic fields with distortions of the black-body spectrum of the cosmic microwave background: Pre- and post-decoupling contributions, *J. Cosmol. Astropart. Phys.* 01 (2014) 009; K. Miyamoto, T. Sekiguchi, H. Tashiro and S. Yokoyama, CMB distortion anisotropies due to the decay of primordial magnetic fields, *Phys. Rev. D* **89**, 063508 (2014); J. Chluba, D. Paoletti, F. Finelli and J. A. Rubino-Martin, Effect of primordial magnetic fields on the ionization history, *Mon. Not. R. Astron. Soc.* **451**, 2244 (2015).
- [15] H. Tashiro, E. Sabancilar, and T. Vachaspati, CMB distortions from superconducting cosmic strings, *Phys. Rev. D* **85**, 103522 (2012); H. Tashiro, E. Sabancilar and T. Vachaspati, CMB distortions from damping of acoustic waves produced by cosmic strings, *J. Cosmol. Astropart. Phys.* 08 **1308**, 035 (2013); H. Tashiro, J. Silk and D.J.E. Marsh, Constraints on primordial magnetic fields from CMB distortions in the axiverse, *Phys. Rev. D* **88**, 125024 (2013); K. E. Kunze and M. Á. Vázquez-Mozo, Constraints on hidden photons from current and future observations of CMB spectral distortions, *J. Cosmol. Astropart. Phys.* 12, (2015) 028; M. Anthonisen, R. Brandenberger, A. Laguë, I. A. Morrison, and D. Xia, Cosmic Microwave Background Spectral Distortions from Cosmic String Loops, *J. Cosmol. Astropart. Phys.* 02 (2016) 047.
- [16] C. M. Will, The Confrontation between general relativity and experiment, *Living Rev. Relativity* **9**, 3 (2006).
- [17] LIGO Scientific and Virgo Collaborations, Test of General Relativity with GW150914, *Phys. Rev. Lett.* **116**, 221101 (2016).
- [18] B. P. Abbott *et al.*, (LIGO Scientific and Virgo Collaborations), Observation of Gravitational Waves from a Binary Black Hole Merger, *Phys. Rev. Lett.* **116**, 061102 (2016).
- [19] A.G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998).
- [20] S. Perlmutter *et al.*, Measurements of  $\Omega$  and  $\Lambda$  from 42 high redshift supernovae, *Astrophys. J.* **517**, 565 (1999).
- [21] G.B. Zhao, T. Giannantonio, L. Pogosian, A. Silvestri, D.J. Bacon, K. Koyama, R.C. Nichol, and Y.S. Song, Probing modifications of General Relativity using current cosmological observations, *Phys. Rev. D* **81**, 103510 (2010).
- [22] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Modified gravity and cosmology, *Phys. Rep.* **513**, 1 (2012).
- [23] K. Koyama, Cosmological tests of modified gravity, *Rep. Prog. Phys.* **79**, 046902 (2016).

- [24] E. G. Adelberger, *Classical Quantum. Gravity.* **18**, 2397 (2001); A. Bauch and S. Weyers, New experimental limit on the validity of local position invariance, *Phys. Rev. D* **65**, 081101 (2002); B. Bertotti, L. Iess, and P. Tortora, A test of general relativity using radio links with the Cassini spacecraft, *Nature (London)* **425**, 374 (2003); S.S. Shapiro, J.L. Davis, D.E. Lebach, and J.S. Gregory, Measurement of the Solar Gravitational Deflection of Radio Waves using Geodetic Very-Long-Baseline Interferometry Data, 1979-1999, *Phys. Rev. Lett.* **92**, 121101 (2004).
- [25] A.A. Abdo *et al.*, A limit on the variation of the speed of light arising from quantum gravity effects, *Nature (London)* **462**, 331 (2009).
- [26] C. Sivaram, *Bull. Astron. Soc. India* **27**, 627 (1999).
- [27] H. Gao, X.F. Wu, and P. Mészáros, Cosmic Transients Test Einstein's Equivalence Principle out to GeV Energies, *Astrophys. J.* **810**, 121 (2015).
- [28] J.J. Wei, H. Gao, X.F. Wu, and P. Mészáros, Testing Einstein's Equivalence Principle With Fast Radio Bursts, *Phys. Rev. Lett.* **115**, 261101 (2015).
- [29] J.J. Wei, J.S. Wang, H. Gao, and X.F. Wu, Tests of the Einstein equivalence principle using TeV blazars, *Astrophys. J. Lett.* **818**, L2 (2016).
- [30] A. Nusser, On testing the equivalence principle with extragalactic bursts, *Astrophys. J. Lett.* **821**, L2 (2016).
- [31] S.N. Zhang, Testing Einstein's equivalence principle with cosmological fast radio bursts behind clusters of galaxies, *Astrophys. J. Lett.* **820**, L31 (2016).
- [32] S. J. Tingay and D. L. Kaplan, Limits on Einstein's equivalence principle from the first localized fast radio bursts FRB 150418, *Astrophys. J. Lett.* **820**, L31 (2016).
- [33] I. Ferreras and I. Trujillo, Testing the wavelength dependence of cosmological redshift down to  $\Delta z \sim 10^{-6}$ , *Astrophys. J.* **825**, 115 (2016).
- [34] J. Magueijo and L. Smolin, Gravity's rainbow, *Classical. Quantum. Gravity.* **21**, 1725 (2004).
- [35] Y. Ling, Rainbow universe, *J. Cosmol. Astropart. Phys.* **8**, 017 (2007).
- [36] Y. Ling and Q. Wu, The big bounce in rainbow universe *Phys. Lett. B* **687**, 103 (2010).
- [37] A. Hees, O. Minazzoli, and J. Larena, Breaking of the equivalence principle in the electromagnetic sector and its cosmological signatures, *Phys. Rev. D* **90**, 124064 (2014).